

## Damping of electromagnetic waves due to electron-positron pair production

S. S. Bulanov,<sup>1,\*</sup> A. M. Fedotov,<sup>2,†</sup> and F. Pegoraro<sup>3,‡</sup>

<sup>1</sup>*Institute of Theoretical and Experimental Physics, 117218 Moscow, Russia*

<sup>2</sup>*Moscow State Engineering Physics Institute, 115409 Moscow, Russia*

<sup>3</sup>*Department of Physics, University of Pisa and INFN, Pisa, Italy*

(Received 12 September 2004; published 11 January 2005)

The problem of the back reaction during the process of electron-positron pair production by a circularly polarized electromagnetic wave propagating in a plasma is investigated. A model based on the relativistic Boltzmann-Vlasov equation with a source term representing the Schwinger formula for the pair creation rate is used. The damping of the wave, the nonlinear up-shift of its frequency due to the plasma density increase, and the effect of the damping on the wave polarization and on the background plasma acceleration are investigated as a function of the wave amplitude.

DOI: 10.1103/PhysRevE.71.016404

PACS number(s): 52.20.-j, 13.40.-f, 12.20.Fv, 52.27.Ep

### I. INTRODUCTION

The production of electron-positron pairs attracts great attention because it is a nonlinear effect that lies beyond the limits of perturbation theory. The study of this effect can shed light on the nonlinear properties of the quantum electrodynamics (QED) vacuum.

This effect was first predicted for the case of a constant electric field more than 60 years ago [1] (see also [2,3]). It is well known that a plane electromagnetic wave cannot produce electron-positron pairs because both its electromagnetic invariants,  $(E^2 - B^2)/2$  and  $\mathbf{E} \cdot \mathbf{B}$ , are equal to zero. For this reason this effect was first considered in the case of a constant electric field, in which case the first invariant  $(E^2 - B^2)/2$  does not vanish. Later, this analysis was extended to the case of a spatially homogeneous time-varying electric field [4–8], but these results were long believed to be of academic interest only, because the power of the laser systems available at that time was far below the limit for pair production to become experimentally observable [6,9,10]. However the recent development of laser technology has resulted in the increase of the power of optical and infrared lasers by many orders of magnitude [11]. Presently, lasers systems are available that can deliver pulses with intensities of the order of  $10^{22}$  W/cm<sup>2</sup> in the focal spot. Such intensities are still much smaller than the characteristic intensity for pair production  $I_{Sch} = 4.6 \times 10^{29}$  W/cm<sup>2</sup>, which corresponds, for a laser pulse with wavelength  $\approx 1$   $\mu$ m, to an electric field equal to the critical Schwinger field  $E_{Sch} = 1.32 \times 10^{16}$  V/cm. Nevertheless, there are projects that aim to reach intensities as high as  $10^{26}$ – $10^{28}$  W/cm<sup>2</sup> already in the coming decade. In addition, several methods for reaching the critical intensity with presently available systems have been proposed recently. One of these schemes was demonstrated in the experiments at SLAC where  $10^{18}$  W/cm<sup>2</sup> laser photons, backscattered by a 46.6 GeV electron beam, interacted

with the laser pulse and several electron-positron pairs were detected [12]. Another scheme for reaching critical intensities was suggested in Ref. [13], where the interaction of the laser pulse with electron density modulations in a plasma, produced by a counterpropagating breaking wake plasma wave, results in the frequency up-shift and pulse focusing. In this scheme intensities of the order of the critical density can be obtained using  $10^{18}$  W/cm<sup>2</sup> laser pulses. Hence, a more detailed study of the Schwinger effect in time-varying electromagnetic fields and of all the processes that accompany it has become an urgent physical problem from an experimental point of view also.

The process of electron-positron pair production by electromagnetic fields that are solutions of the Maxwell equations in a plasma and in vacuum was studied recently in Refs. [14,15]. In Ref. [15] it was shown that already for intensities of the laser pulse smaller than the critical intensity, the energy loss due to pair production is of the same order of the energy stored in the pulse. Therefore it is no longer possible to consider the electromagnetic field in the pulse as an external field and the energy loss by the electromagnetic field due to pair production and particle acceleration must be taken into account.

The problem of the backreaction of the produced particles on the background field was discussed extensively in a number of papers on the particle formation process in high energy hadronic interactions [16–19] as well as under the action of electric fields [20,21]. In the former case this process can be viewed as the quantum tunneling of quark-antiquark and gluon pairs in the presence of the background color-electric field of quantum chromodynamics (QCD). Such a color field is formed between two receding nuclei which are color charged by the exchange of soft gluons at the time of collision. This leads to the formation of a very strong color-electric field and, hence, to more copious pair production. It was understood that, in solving a dynamical problem with a strong initial electric field, the effect of the produced particles on the electric field (the back reaction) should be taken into consideration. The quark-gluon plasma emerging through tunneling in nucleus-nucleus collisions will change the color-electric field due to the appearance of conduction and polarization currents. The first current is due to the par-

\*Electronic address: bulanov@heron.itep.ru

†Electronic address: fedotov@cea.ru

‡Electronic address: pegoraro@df.unipi.it

particle motion in the field while the latter arises from the process of pair creation. A kinetic equation coupled to Maxwell equations was used to solve this problem. The pair production process was considered as if occurring in QED, and the color-electric field was assumed to be Abelian, spatially homogeneous and time-dependent. However a spatially homogeneous time dependent electric field is not a solution of Maxwell equations in vacuum. We also note that in Refs. [16–21] special attention was paid to the properties of the emerging plasma, while the properties of the background field were not studied in detail.

In the present paper, we consider the process of electron-positron pair production in a cold collisionless plasma, under the action of an electromagnetic field which is an actual solution of the Maxwell equations, as well as the backreaction of the produced pairs on the background field. In doing so we use the Boltzmann-Vlasov equation, with a source term obtained from the pair production rate [16–18]. In order to elucidate the role of the magnetic field component on the electron-positron pair production, we consider a planar, circularly polarized, electromagnetic wave propagating in an underdense collisionless plasma (for the sake of simplicity we consider an electron-positron plasma). In the case of a plane wave in a plasma the first field invariant  $(E^2 - B^2)/2$  is not equal to zero due to the different dispersion equation with respect to that in vacuum. Therefore, in a plasma, electron-positron pairs can be produced by a plane electromagnetic wave, as was shown in Ref. [14]. In this case a Lorentz transformation to the reference frame moving with the group velocity  $v_g$  of the wave transforms the electromagnetic field into a purely electric field, that rotates with constant frequency, and with no associated magnetic field. Although this transformation reduces the problem under consideration to the situation where the pairs are produced by a time-varying electric field, the effects of the wave magnetic field are incorporated rigorously into our model. We notice that a similar approach was used earlier in Ref. [22]. However in this latter paper a linearly polarized electromagnetic wave was considered and an approximation was adopted that is only valid in the limit of a small amplitude electromagnetic field.

In the present paper we consider the effect on the background wave in the plasma caused by the pairs produced by the wave through their polarization and conduction currents. In particular, by considering the interaction between the wave and the plasma as an initial value problem in the moving frame, we study the evolution of the wave electromagnetic field. Due to the nonlinear properties of the equations governing the field evolution in time, we find a strong nonlinear dependence of wave field properties on the wave initial amplitude. We find a nonlinear up-shift of the wave frequency, a change of its polarization state and damping of its amplitude. In order to exemplify these effects, we consider two limiting cases of the electric field evolution. In the first limit the amplitude of the initial electric field is assumed to be so large that most of the electron-positron pairs are produced instantaneously. This leads to an instantaneous change of the electric field amplitude and frequency. In the second limit we consider a regime where the pair production rate is relatively small, so that all parameters of the wave change

slowly. In this case we see the wave amplitude damp with time.

This paper is organized as follows. In Sec. II we review some well known properties of a strong electromagnetic wave in a plasma. In Sec. III we study the equations governing the process of pair production and the evolution of the electromagnetic wave. In Secs. IV and V we consider two limiting cases: fast changing field and slow changing field. The main results and conclusions are presented in Sec. VI.

## II. RELATIVISTICALLY STRONG ELECTROMAGNETIC WAVE IN A PLASMA

First, we recover the properties of a relativistically strong electromagnetic wave propagating in an underdense collisionless electron-positron plasma that are needed in order to determine the form of the time varying electric field. This discussion is based on the results obtained by Akhiezer and Polovin in Ref. [23]. We consider a circularly polarized electromagnetic wave, propagating in an electron-positron plasma. In the following we use the  $c=1$  and  $\hbar=1$  convention.

An electromagnetic wave propagating in a plasma has a group velocity smaller than the speed of light in vacuum. Thus it is possible to make a transformation to the reference frame moving with the wave group velocity  $v_g$ . In this frame the magnetic field of the wave vanishes and its time-varying electric field is spatially homogeneous and is thus governed by the equation

$$\frac{d\mathbf{E}}{dt} = -4\pi\mathbf{j} = -4\pi \sum_{\alpha=+,-} \frac{e_\alpha}{(2\pi)^3} \int \mathbf{v} f_\alpha(\mathbf{p}, t) d^3p, \quad (1)$$

where  $\mathbf{v} = \mathbf{p}/(m^2 + p^2)^{1/2}$ ,  $p = |\mathbf{p}| = (p_x^2 + p_y^2 + p_z^2)^{1/2}$ ,  $f_\alpha(\mathbf{p}, t)$  is the positron (electron) distribution function, normalized such that  $\int f_\alpha(\mathbf{p}, t) d^3p / (2\pi)^3 = n_\alpha$  gives the number  $n_\alpha$  of electrons or positrons per unit volume, and  $e_\alpha$  is their electric charge with  $\alpha=+$  for the positrons and  $\alpha=-$  for the electrons. We consider the case of an electrically quasineutral plasma, where the density of positrons is equal to the density of electrons,  $n_+ = n_- = n_0$ , with  $n_0$  the density in the moving frame. If we assume that the plasma is cold, i.e., that the particle distribution function of the species  $\alpha$  can be written as  $f_\alpha(\mathbf{p}, t) = n_\alpha (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}_\alpha(t))$ , the system of equations for the electric field evolution in the moving frame reduces to

$$\frac{d\mathbf{E}}{dt} = -4\pi n_0 \sum_{\alpha=+,-} e_\alpha \mathbf{v}_\alpha, \quad (2)$$

$$\frac{d\mathbf{p}_\alpha}{dt} = e_\alpha \mathbf{E}, \quad \text{with } \mathbf{v}_\alpha = \frac{\mathbf{p}_\alpha}{(m^2 + p_\alpha^2)^{1/2}}. \quad (3)$$

We assume symmetric initial conditions, so that the positron and the electron momenta  $p_{\pm,\perp}$  perpendicular to the direction of propagation of the laser pulse have opposite signs and equal absolute value,  $\mathbf{p}_{+,\perp} = -\mathbf{p}_{-,\perp} = \mathbf{p}_\perp$ , while they have equal parallel momentum  $p_{\parallel} = p_{\parallel,0}$ . Here the nonzero value  $p_{\parallel,0} = -mv_g \gamma_g$  of the parallel momentum is due to the Lorentz transformation from the laboratory to the moving frame and

$\gamma_g = (1 - v_g^2/c^2)^{-1/2}$ . For these initial conditions and for a circularly polarized pulse, the square of the electron momentum  $p^2 = |\mathbf{p}|^2 = (p_x^2 + p_y^2 + p_z^2)$  is constant in time and the solution of the equation of motion

$$\frac{d^2 \mathbf{p}_\perp}{dt^2} = - \frac{8\pi e^2 n_0 \mathbf{p}_\perp}{(m^2 + p^2)^{1/2}} \quad (4)$$

for the particle perpendicular momentum  $p_{\pm,\perp}$ , which is obtained by combining Eqs. (2) and (3), is simply given by

$$\mathbf{p}_\perp = -P(\mathbf{e}_x \cos \Omega t + \mathbf{e}_y \sin \Omega t),$$

$$\mathbf{A} = A_0(\mathbf{e}_x \cos \Omega t + \mathbf{e}_y \sin \Omega t), \quad (5)$$

where  $\mathbf{A}$  is the vector potential,  $A_0 = P/e$  and

$$\Omega = \left[ \frac{8\pi e^2 n_0}{(m^2 + P^2 + p_{\parallel,0}^2)^{1/2}} \right]^{1/2} \quad (6)$$

is the Langmuir frequency which enters the dispersion equation of an electromagnetic wave propagating in a plasma. In this moving frame the wave electric field is given by

$$\mathbf{E} = \Omega A_0(\mathbf{e}_x \sin \Omega t - \mathbf{e}_y \cos \Omega t). \quad (7)$$

Note that, since in the moving frame the wave number of the oscillating laser field is zero, its frequency coincides with the Langmuir frequency. In the laboratory frame instead the wave dispersion equation is given by

$$\omega^2 = k^2 + \Omega^2, \quad (8)$$

where  $\Omega$  can be reexpressed in terms of the electron and positron densities  $n = n_0/\gamma_g$  and of the particle energy  $(m^2 + p^2)^{1/2}$  in the laboratory frame as  $\Omega = [(8\pi e^2 n)/(m^2 + p^2)^{1/2}]^{1/2}$ . The phase and group velocities of a nonlinear electromagnetic wave in a plasma depend on the plasma parameters and on the wave amplitude. From Eq. (8) we find that the phase velocity  $v_{ph} = \omega/k$  and the group velocity  $v_g = \partial\omega/\partial k$ , are related according to expression  $v_{ph}v_g = 1$ . In the laboratory frame the electric and the magnetic fields are given by

$$\mathbf{E} = -\partial_t \mathbf{A} = \omega A_0[\mathbf{e}_x \sin(\omega t' - kx) - \mathbf{e}_y \cos(\omega t' - kx)], \quad (9)$$

$$\mathbf{B} = \nabla \times \mathbf{A} = k A_0[\mathbf{e}_x \cos(\omega t' - kx) - \mathbf{e}_y \sin(\omega t' - kx)], \quad (10)$$

respectively, with  $t'$  the time in the laboratory frame.

We see that in a plasma the first invariant of the electromagnetic field  $\mathcal{F} = (E^2 - B^2)/2$  is not equal to zero and is given by

$$\mathcal{F} = \frac{\Omega^2}{2} A_0^2 \equiv \frac{1}{2} \left( \frac{\Omega}{\omega} \right)^2 E_0^2. \quad (11)$$

It vanishes when the plasma density tends to zero, i.e., in vacuum. In the following we shall use the notation  $E = (\Omega A_0) \equiv (\Omega/\omega) E_0$ .

In Ref. [22] the model case of a linearly polarized wave was considered. However a finite amplitude linearly or elliptically polarized wave propagating in a plasma has both

transversal and longitudinal components, which oscillate at different frequencies (see, e.g., Ref. [24]). Thus the problem of the pair production by a linearly polarized wave is more complicated to solve.

### III. KINETIC DESCRIPTION OF THE ELECTRON-POSITRON PAIR PLASMA

We consider the propagation of a circularly polarized electromagnetic wave in an underdense collisionless plasma in the reference frame moving with the wave group velocity  $v_g$ . The relativistic kinetic equation

$$\frac{\partial f_\alpha}{\partial t} + e_\alpha \mathbf{E} \cdot \frac{\partial f_\alpha}{\partial \mathbf{p}} = q_\alpha(E, p), \quad (12)$$

describes the dependence on time and momentum of the distribution function  $f_\alpha(\mathbf{p}, t)$  in this moving frame where a spatially homogeneous electric field  $\mathbf{E}$  is present. The source term in Eq. (12) is proportional to the quasiclassical probability

$$\exp \left[ - \frac{\pi(m^2 + p_\perp^2)}{|e\mathbf{E}(t)|} \right] \quad (13)$$

of tunneling through the gap between the lower and the upper continuum of electron energy spectrum in the presence of the electric field. We note that the form of Eq. (13) corresponds to the case of constant field [2]. However, the characteristic time of pair production  $c/l_c$ , where  $l_c = \hbar/mc$  is electron Compton wavelength, is negligible with respect to the wave period. This estimate gives a lower bound on the pair production time while the estimate that follows from the quasiclassical approximation gives for the time of sub-barrier motion  $t_{tun} = 1/a\omega$  [7,14], where  $a = eA/mc$  is the dimensionless amplitude of vector potential. However, for  $a \gg 1$  (the case we are considering), even this estimate yields a pair production time much shorter than the wave period. Thus, it is possible to use Eq. (13) for the time-varying electric field with time playing the role of a parameter. In addition, following the reasoning of Refs. [16–18], we assume that the pairs are produced at rest, i.e., the momentum distribution of the source term is taken to be proportional to the Dirac delta function

$$q_\alpha(E, p) = \frac{e^2 \mathbf{E}(t)^2}{2} \exp \left[ - \frac{\pi m^2}{|e\mathbf{E}(t)|} \right] \delta(\mathbf{p}). \quad (14)$$

This assumption is reinforced by the fact that the momentum distribution in Eq. (13) has a width  $p_\perp \sim [e\mathbf{E}(t)]^{1/2} = m[e\mathbf{E}(t)]^{1/2} \ll m$  which is negligible with respect to the momentum that electrons (positrons) acquire in the electric field. Here  $\eta = e\mathbf{E}/m^2 = \mathbf{E}/E_{Sch} \ll 1$  is the normalized electric field and  $E_{Sch} = m^2/e$  is the critical Schwinger field. The source term has been normalized in such way that  $\int q_\alpha(E, p) d^3p / (2\pi)^3$  gives the number  $[(eE)^2/4\pi^3] \times \exp[-\pi m^2/eE]$  of positrons (electrons) produced according to Schwinger's formula.

We solve Eq. (12) by integrating it along the particle characteristics. The equations for the characteristics for each species  $\alpha$  are

$$p_{\parallel} = p_{\parallel,0}, \quad \frac{dp_{\perp}}{dt} = e_{\alpha} E, \quad \frac{df_{\alpha}}{dt} = q_{\alpha}. \quad (15)$$

Introducing the function  $\mathbf{A}(t) = -\int_0^t \mathbf{E}(s) ds$  by integrating  $\mathbf{E}(t)$  over time, we obtain

$$\mathbf{p}_{\perp} = e_{\alpha} \int_0^t \mathbf{E}(s) ds + \mathbf{p}_{\perp,0}, \quad \text{i.e.,} \quad \mathbf{p}_{\perp} + e_{\alpha} \mathbf{A}(t) = \mathbf{p}_{\perp,0}. \quad (16)$$

As a result the distribution function is given by the following expression

$$f_{\alpha} = f_{\alpha,0}(p_{\parallel}, \mathbf{p}_{\perp} + e_{\alpha} \mathbf{A}(t)) + \int_0^t q_{\alpha} \{ \mathbf{p}_{\perp} + e_{\alpha} [\mathbf{A}(t) - \mathbf{A}(t')], t' \} dt', \quad (17)$$

where  $f_{\alpha,0}(p_{\parallel}, \mathbf{p}_{\perp})$  is the distribution function of the initial plasma positrons (electrons) before the passage of the electromagnetic wave. Let us assume that at the initial time  $t=0$  the plasma is cold so that

$$f_{\alpha,0} = n_0 (2\pi)^3 \delta(p_{\perp}) \delta(p_{\parallel} - p_{\parallel,0}), \quad (18)$$

where we recall that  $p_{\parallel}$  is the component of the particle momentum parallel to the electromagnetic wave propagation and  $p_{\parallel,0}$  is its initial value which arises from the Lorentz transformation from the laboratory to the moving frame.

The modification of the kinetic equation given by the source term in Eq. (14) must also be accompanied by a change of the source term in Maxwell equations. The electron-positron pair production by a spatially homogeneous time-varying electric field leads to the appearance of a time-dependent electric dipole which generates a polarization current. Thus the current density in Eq. (1) acquires an additional term with respect to the situation when no pair production is present. Then Eq. (2) reads as [18]

$$\frac{d\mathbf{E}}{dt} = -4\pi \mathbf{j}_{tot} = -4\pi (\mathbf{j}_{cond} + \mathbf{j}_{pol}), \quad (19)$$

where the conduction current is

$$\mathbf{j}_{cond}(t) = e \sum_{\alpha=+,-} \int f_{\alpha}(\mathbf{p}, t) \frac{\mathbf{p}}{(m^2 + p^2)^{1/2}} \frac{d^3 p}{(2\pi)^3}, \quad (20)$$

and the polarization current is [16] (see the Appendix for details)

$$\mathbf{j}_{pol}(t) = \frac{\mathbf{E}(t)}{|\mathbf{E}(t)|^2} \sum_{\alpha=+,-} \int q_{\alpha}(\mathbf{p}, t) (m^2 + p^2)^{1/2} \frac{d^3 p}{(2\pi)^3}. \quad (21)$$

Using the distribution function (17), we obtain the following expressions for the current densities

$$\begin{aligned} \mathbf{j}_{cond}(t) &= -2e^2 n_0 \frac{\mathbf{A}(t)}{[m^2 + \mathbf{p}_{\parallel,0}^2 + e^2 A^2(t)]^{1/2}} \\ &\quad - 2e^2 \int_0^t \frac{\mathbf{A}(t) - \mathbf{A}(t')}{[m^2 + e^2 |\mathbf{A}(t) - \mathbf{A}(t')|^2]^{1/2}} \frac{|e\mathbf{E}(t')|^2}{8\pi^3} \\ &\quad \times \exp\left[-\frac{\pi m^2}{|e\mathbf{E}(t')|}\right] dt', \end{aligned} \quad (22)$$

$$\mathbf{j}_{pol}(t) = \frac{e^2 m}{2\pi^2} \mathbf{E}(t) \exp\left[-\frac{\pi m^2}{|e\mathbf{E}(t)|}\right], \quad (23)$$

where  $A = |\mathbf{A}(t)|$ . In performing the momentum space integration we have used the fact that the pairs are produced with zero momentum. Inserting these expressions for the current densities into the right-hand side of Eq. (19) and using the dimensionless vector potential  $\mathbf{a} = e\mathbf{A}/m$  and the normalized electric field  $\eta = e\mathbf{E}/m^2$ , we obtain the equation for the electric field evolution in the presence of pair production

$$\frac{d\mathbf{a}(t)}{dt} = -m\eta(t),$$

$$\begin{aligned} m \frac{d\eta(t)}{dt} &= \omega_p^2 \frac{\mathbf{a}(t)}{[1 + \tilde{p}_{\parallel,0}^2 + a^2(t)]^{1/2}} \\ &\quad + \kappa \int_0^t \frac{\mathbf{a}(t) - \mathbf{a}(t')}{[1 + |\mathbf{a}(t) - \mathbf{a}(t')|^2]^{1/2}} \frac{|\eta(t')|^2}{8\pi^3} \\ &\quad \times \exp\left[-\frac{\pi}{|\eta(t')|}\right] dt' - \frac{em^3}{2\pi^2} \eta(t) \exp\left[-\frac{\pi}{|\eta(t)|}\right], \end{aligned} \quad (24)$$

where  $\omega_p = (8\pi e^2 n_0/m)^{1/2}$  is the nonrelativistic Langmuir frequency,  $\tilde{p}_{\parallel,0} \equiv p_{\parallel,0}/m$  and  $\kappa = 8\pi e^2 m^4$ , where the factor  $m^4$  stands for the inverse of the invariant Compton 4-volume  $m^4 = c/t_c^4 \approx 0.14 \times 10^{53} \text{ cm}^{-3} \text{ s}$ . Similar equations were obtained in Refs. [18], where however there was no initial distribution function and a spatially homogeneous electric field in vacuum was used, which is not a solution of Maxwell's equations.

The nonlinear integro-differential equation (24) cannot be solved analytically. Numerical solutions of this equation are presented in Fig. 1 for different initial amplitudes. We can see that the process of electron-positron pair production leads to the damping of the wave in the plasma and to the nonlinear up-shift of its frequency. The damping is due to the fact that each pair creation takes a portion of the field energy equal to  $2mc^2$  as well as the amount needed for the particle acceleration. The up-shift of the field frequency is due to the increase of the plasma density, and thus of the Langmuir frequency, as new pairs are created. This frequency up-shift is seen in Fig. 1, and bears a strong resemblance to the blue-shift of an electromagnetic wave that propagates in a medium that becomes ionized, as investigated theoretically in Refs. [25] and experimentally in Refs. [26].

Since the pair production rate depends on the field amplitude exponentially, an unbalanced damping of the field



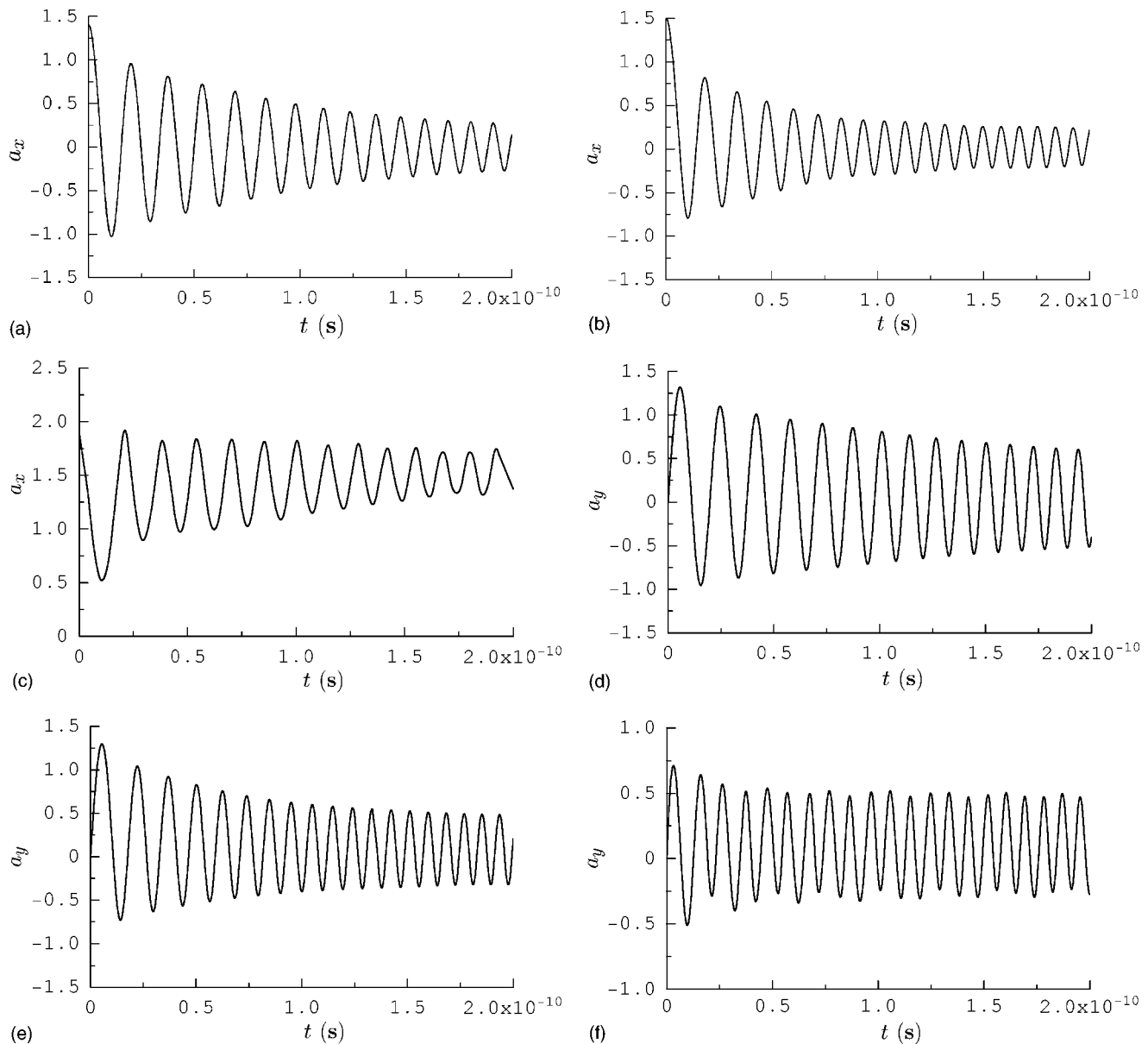


FIG. 1. Time evolution in the moving frame of the  $x$  and the  $y$  components of the dimensionless vector potential for different initial amplitudes:  $a=1.4 \times 10^5$  (a), (d),  $a=1.5 \times 10^5$  (b), (e),  $a=1.9 \times 10^5$  (c), (f) with initial plasma density  $n_0=10^{19} \text{ cm}^{-3}$  in the moving frame;  $v_g \approx 1$ ;  $\gamma_g=10$ . The upper row shows the  $x$  component of the vector potential, the lower the  $y$  component. On the  $x$  axis time is measured in seconds;  $a=1$  corresponds, for a  $1 \mu\text{m}$  wavelength pulse, to an intensity of  $10^{18} \text{ W/cm}^2$  and  $a=4.6 \times 10^5$  to the Schwinger intensity.

components can occur and lead to a change of the field polarization. We should also note that the decrease of the amplitude of the vector potential is accompanied by a frequency up-shift, so that the decrease of the amplitude of the electric field is not as fast as that of the vector-potential. These properties of the electric field are shown in Fig. 2, where the projections of the polarization vector are presented for the same set of initial parameters as in Fig. 1. In Fig. 2(a) we see the damping of the  $x$  component of the electric field and the transition from circular to elliptical polarization with the major axis of the ellipse directed along the  $y$  axis. In addition, in Fig. 2(b) we see a rotation of the principal axes of the ellipse. The rotation of the principal axes of the polarization ellipse was discussed in Ref. [24] in the case of the free propagation

(without source terms) of an elliptically polarized nonlinear pulse in a plasma. The situation shown in Fig. 2(c) is different from the two previous ones. In this latter case the pair production rate at the beginning of the field evolution is so large [see Fig. 3(c)] that the first wave oscillation cycle cannot be completed, leading to oscillations of the  $x$  component of the wave vector potential around a nonzero mean value determined by the balance between the time averaged parts of the first two terms on the right-hand side of the second of Eqs. (24). This shift of the center of the oscillations of the  $x$  component of the vector potential leads to a reduction of the oscillation frequency of this wave component so that, in this case, the  $x$  and the  $y$  components of the wave oscillate at different frequencies.

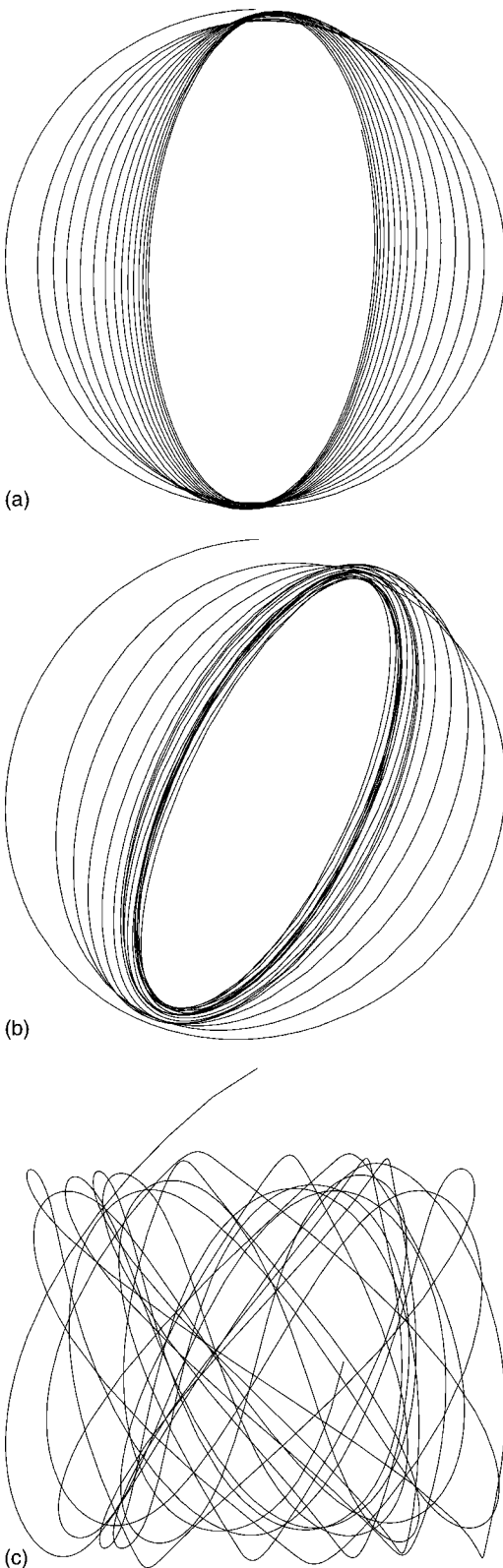


FIG. 2. Trajectories of the projections of the electric field polarization vector for the same set of initial conditions as in Fig. 1.

The increase with time of the pair plasma density is shown in Fig. 3 starting from an initial density before the start of the pulse propagation. We see that the particle density increases in steps as a function of time. This time depen-

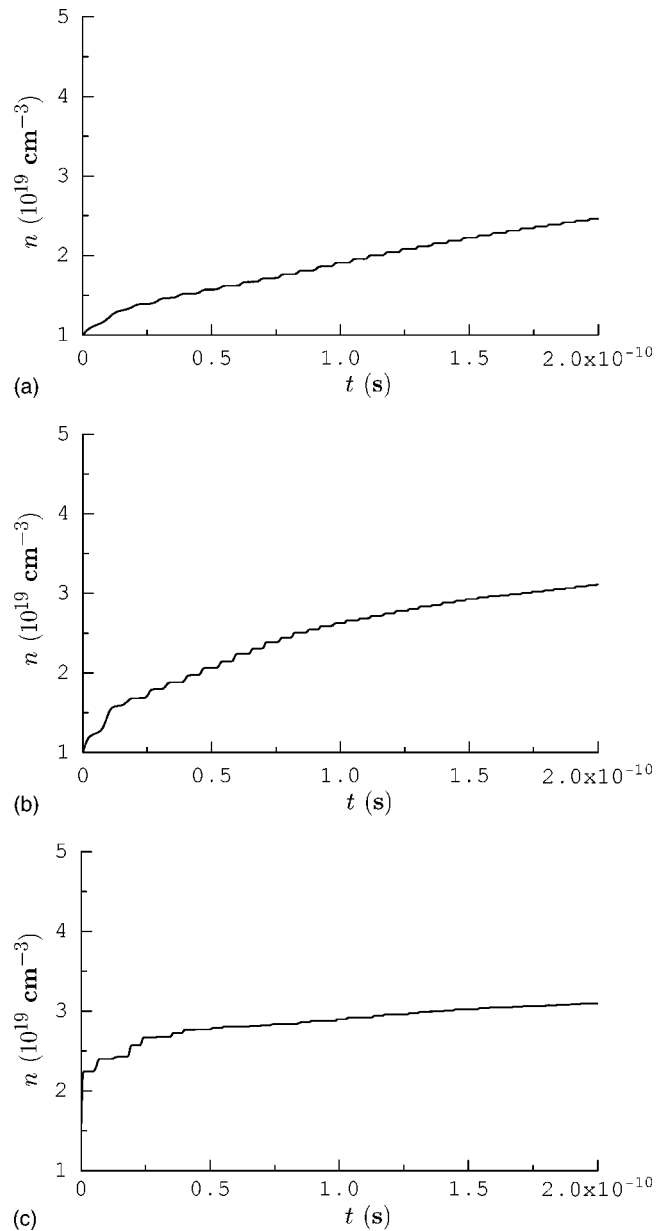


FIG. 3. Dependence of the plasma electron (positron) density on time for the same set of initial conditions as in Fig. 1 in the moving frame:  $a=1.4 \times 10^5$  (a),  $a=1.5 \times 10^5$  (b), and  $a=1.9 \times 10^5$  (c). The density is measured in units of  $10^{19} \text{ cm}^{-3}$  and time in seconds.

dence is due to the oscillations of the amplitude of the electric field that occurs in an elliptically polarized wave (see Fig. 2). The electron-positron pairs are mainly produced near the maxima of the electric field amplitude, while there is practically no pair production in between. From these plots we can also see that the frequency is up-shifted since the width of the steps decreases with time. For the pulse amplitude of frame (c), the pair production occurs almost instantaneously at the beginning of the pulse evolution.

The difference between the above three cases is clearly illustrated by the different shapes of the particle distribution functions in the  $p_x$ - $p_y$  plane. (Note that the electron and the positron distributions are one the mirror image of the other.) In cases (a) and (b) electrons and positrons are mostly cre-

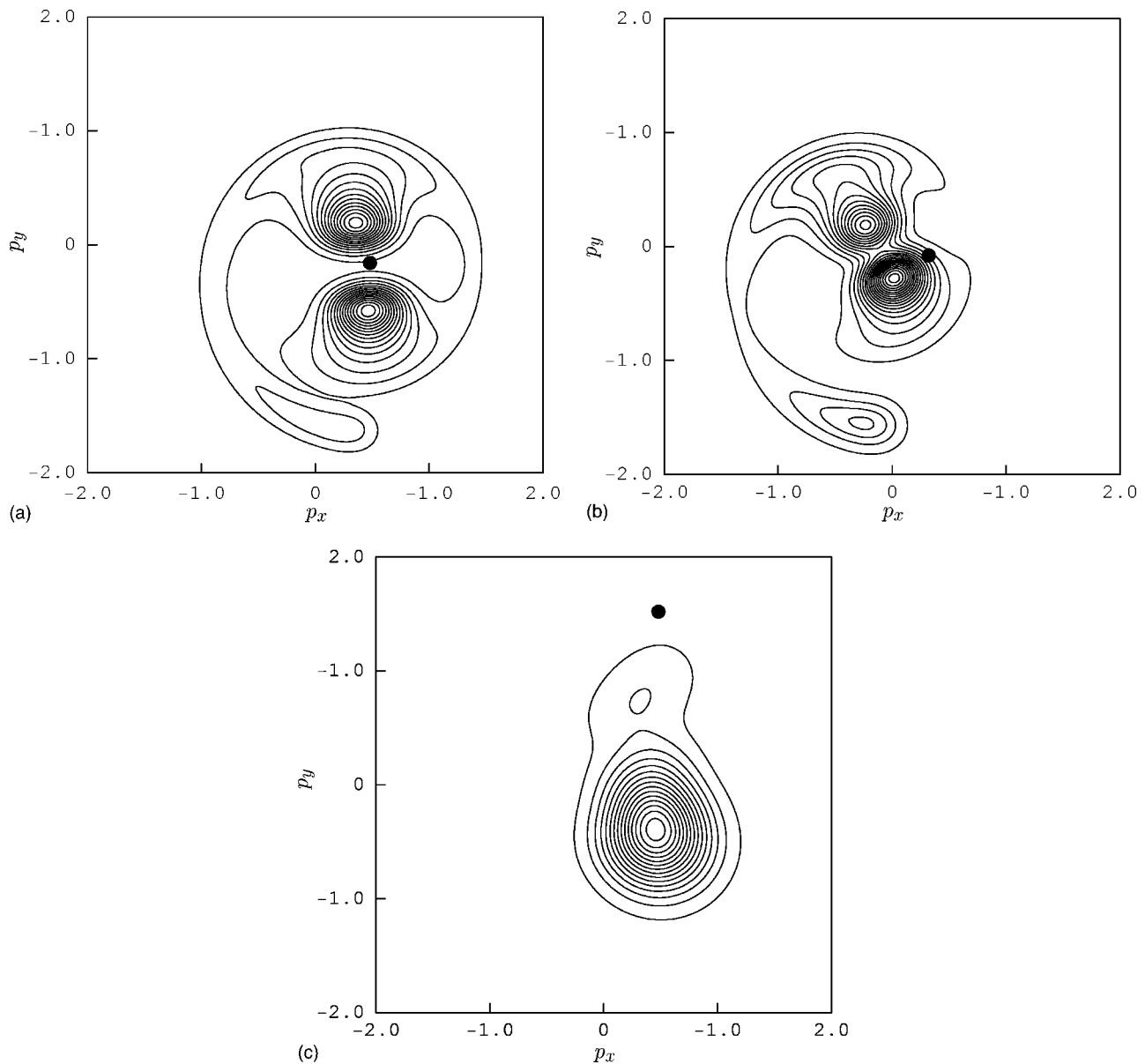


FIG. 4. The electron distribution functions versus  $p_x$  and  $p_y$  in the moving reference frame for the same set of initial conditions as in Fig. 1 at time  $2 \times 10^{-10}$  s. Particle momenta are normalized on the dimensionless vector-potential amplitude  $a$  multiplied times  $10^5$ . Black circles correspond to the initial plasma particle distribution at time  $2 \times 10^{-10}$  s.

ated at the maxima of the electric field  $|\mathbf{E}|$  (and thus of the vector potential  $|\mathbf{A}|$ ). Since at birth  $\mathbf{p}_\perp = \mathbf{0}$ , in the case of a circularly polarized electric field this should lead to a ring type distribution. However, since the wave polarization becomes elliptical because of the back reaction due to the pair creation, the distribution function of each population consists, in the canonical momentum  $\mathbf{p}_\perp + e_a \mathbf{A}$  plane, mainly of two blobs at  $\pm e_a \mathbf{A}_{\max}$ . In the  $p_x$ - $p_y$  plane, these blobs move according to the time evolution of the vector potential  $\mathbf{A}$ . On the contrary the position of the initial distribution function (denoted by a dark dot in the figure) corresponds to  $\mathbf{p}_\perp + e_a \mathbf{A} = \mathbf{0}$ . In case (c) the pairs are created mostly at the start at  $\mathbf{p}_\perp + e_a \mathbf{A} = e_a \mathbf{A}(t=0)$ . Since the time evolution of  $\mathbf{A}(t)$  is ergodic, as shown in Fig. 2, their distribution tends to be randomized in the  $p_x$ - $p_y$  plane. (See Fig. 4(c).)

The particle distribution function is shown in Fig. 5 versus the parallel momentum  $p_\parallel$  in the laboratory frame at time  $t = 2 \times 10^{-10}$  s. Note that in case (c) the strong damping of the wave due to the pair creation and the resulting nonadiabatic interaction has led to a strong acceleration of the particles in the initial plasma. Such large values of the longitudinal momentum of electrons (positrons) in the laboratory frame are due to the transverse acceleration of electrons (positrons) in the moving frame. Performing the Lorentz transformation back to the laboratory frame we obtain for the longitudinal momentum in the laboratory frame of the initial electrons and positrons  $p_\parallel = \gamma_g [p_{\parallel o} + v_g (1 + p_{\perp o}^2 + |\mathbf{a}^2|)^{1/2}] \approx \gamma_g v_g |\mathbf{a}|$  where we used  $|\mathbf{a}| \gg |p_{\perp o}|$ .

In summary, the production of electron-positron pairs by the electromagnetic wave propagation in the plasma leads to

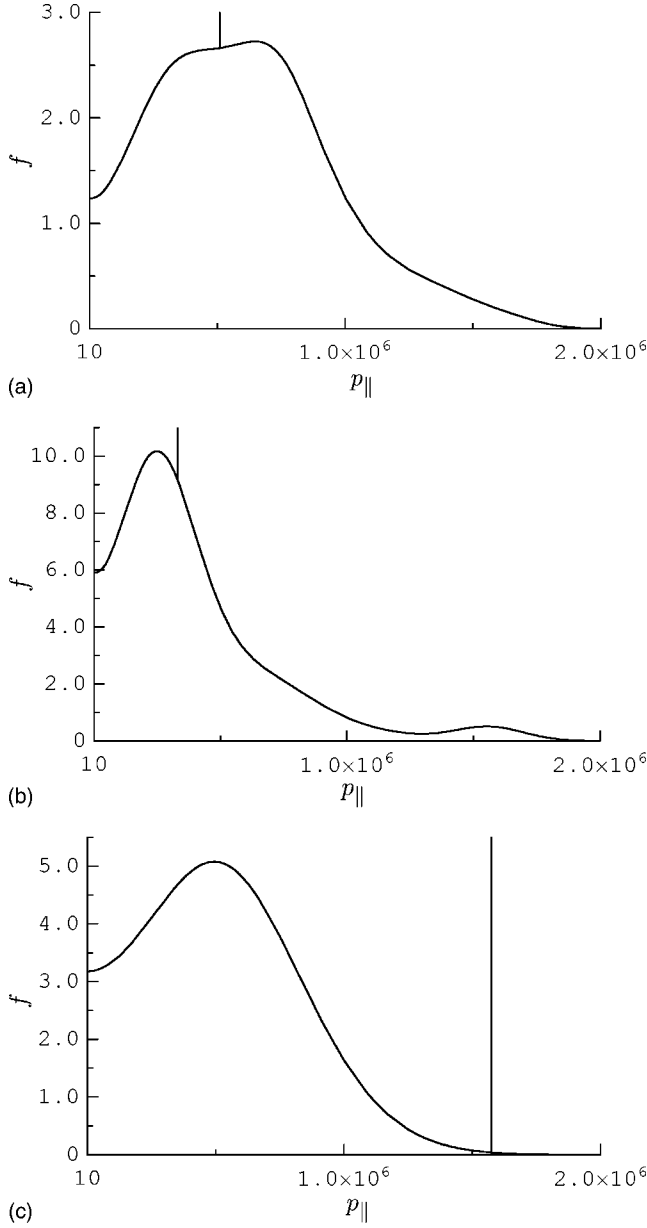


FIG. 5. The electron distribution functions versus  $p_{\parallel}$  in the laboratory reference frame for the same set of initial conditions as in Fig. 1 at time  $2 \times 10^{-10}$  s. Momentum is measured in the same units as in Fig. 4. The vertical lines correspond to the distribution of the initial plasma particles.

the up-shifting of the wave frequency and to the damping of the wave amplitude and changes the polarization state of the wave. In order to illustrate these effects analytically, we consider two limiting cases of the electric field evolution, which can be clearly distinguished on the basis of the results shown in Fig. 1. First, we will consider the case of a fast changing field when the initial electric field amplitude is so large that the most of the electron-positron pairs is produced instantly. This case can be illustrated by the results shown Fig. 1(c). Then, we will consider the case of a slowly changing field, when the pair production rate is relatively small and all the parameters of the wave change slowly as illustrated in Fig. 1(a).

#### IV. FAST CHANGING ELECTROMAGNETIC FIELD

We assume that the amplitude of the initial electric field is so large that the electron-positron pair production and the consequent change of the properties of the wave electric field occur almost instantaneously. In this case we may approximate the time dependence of the source term as

$$q_{\alpha}(E, p, t) = \tau \delta(t) q_{\alpha}(E_0, p), \quad (25)$$

where  $E_0$  is the initial amplitude of the electric field and  $\tau$  is a characteristic time defined such that the total number of pairs produced is kept constant and given by  $\tau(|\eta_0|^2/8\pi^3)\exp[-\pi/|\eta_0|]$  per Compton 3-volume  $l_c^3$ . Then, it is possible to carry out the integration over  $t'$  in the right-hand side of Eq. (24). As a result we obtain

$$\begin{aligned} \frac{d^2 \mathbf{a}}{dt^2} + \omega_p^2 \frac{\mathbf{a}(t)}{[1 + p_{\parallel 0}^2 + a^2(t)]^{1/2}} \\ = -\kappa \tau \frac{|\eta_0|^2}{8\pi^3} \exp\left[-\frac{\pi}{|\eta_0|}\right] \frac{\mathbf{a}(t) - \mathbf{a}_0}{[1 + |\mathbf{a}(t) - \mathbf{a}_0|^2]^{1/2}}. \end{aligned} \quad (26)$$

We note that  $\mathbf{a}_0$  in Eq. (26) and the initial value  $\mathbf{a}(0)$  of the amplitude of the vector potential in Eq. (26) are connected in an indirect way since the initial conditions for Eq. (26) are determined by the value of the electric field after the main part of the pair production has already taken place when the amplitude of the field has been reduced so that it is no longer capable of producing a significant amount of pairs. Therefore the values  $\mathbf{a}_0$ ,  $\eta_0$  that enter the right-hand side of Eq. (26) should be considered as referring to the vector potential and to the electric field at the instant when the pairs are created. From these values we can deduce the value of  $\mathbf{a}(0)$  using energy conservation and taking into account the polarization current.

In order to understand the basic properties of the electric field evolution described by Eq. (26), it is convenient to linearize it. To do so, we assume that  $a \ll 1$  and obtain

$$\mathbf{a}''(t) + (\omega_p'^2 + \kappa N)\mathbf{a}(t) - \kappa N\mathbf{a}_0 = 0, \quad (27)$$

where  $N = \tau[|\eta_0|^2/8\pi^3]\exp[-\pi/|\eta_0|]$  and  $\omega_p'^2 = \omega_p^2/(1 + p_{\perp 0}^2)^{1/2}$ . We see that Eq. (27) describes oscillations with frequency  $(\omega_p'^2 + \kappa N)^{1/2}$ . Taking  $\mathbf{a}_0 = (a_{0z}, 0)$  we obtain

$$\begin{aligned} a_x(t) = \frac{a_{0x}\kappa N}{\omega_p'^2 + \kappa N} - \frac{a_{0x}\kappa N}{\omega_p'^2 + \kappa N} \cos[(\omega_p'^2 + \kappa N)^{1/2}t] \\ + a_1 \cos[(\omega_p'^2 + \kappa N)^{1/2}t], \end{aligned} \quad (28)$$

$$a_y(t) = -a_1 \sin[(\omega_p'^2 + \kappa N)^{1/2}t], \quad (29)$$

where the initial conditions are  $\mathbf{a}(0) = (a_1, 0)$ ,  $\mathbf{a}'(0) = (0, (\omega_p'^2 + \kappa N)^{1/2}a_1)$ . Thus, Eqs. (28) and (29) describe an elliptically polarized field oscillating with an upshifted frequency, as consistent with the numerical solution of Eq. (24).

#### V. SLOWLY CHANGING ELECTROMAGNETIC FIELD

We assume that the pair production rate is small, so that the parameters of the electromagnetic wave change slowly.



Then, we can neglect  $a(t)$  with respect to  $a(t')$ , since, as the wave amplitude decreases with time,  $a(t)$  becomes increasingly small compared to  $a(t')$ . Then, after differentiation with respect to time, Eq. (24) reads

$$\begin{aligned} \frac{d^3 \mathbf{a}(t)}{dt^3} + \omega_p^2 \frac{d}{dt} \frac{\mathbf{a}(t)}{[1 + p_{\parallel 0}^2 + \mathbf{a}^2(t)]^{1/2}} \\ = \kappa \frac{\mathbf{a}(t)}{[1 + a^2(t)]^{1/2}} |\eta(t)|^2 \exp\left[-\frac{\pi}{|\eta(t)|}\right]. \end{aligned} \quad (30)$$

If we linearize Eq. (30) and set the pair production rate equal to constant, we obtain

$$\mathbf{a}'''(t) + \frac{\omega_p^2}{(1 + p_{\parallel 0}^2)^{1/2}} \mathbf{a}'(t) - \kappa W \mathbf{a}(t) = \mathbf{0}, \quad (31)$$

where  $W = |\bar{e}|^2 \exp(-\pi/|\bar{e}|)$  and  $\bar{e}$  is the electric field amplitude averaged over the field evolution. The equations for the two components of the vector potential are decoupled and thus the polarization state is preserved. The solution of Eq. (30) is

$$\begin{aligned} a_{x,y}(t) = C_{1,x,y} \exp(-i\omega_1 t) + C_{2,x,y} \exp(-i\omega_2 t) \\ + C_{3,x,y} \exp(-i\omega_3 t), \end{aligned} \quad (32)$$

where the frequencies  $\omega_i$  ( $i=1,2,3$ ) are the roots of the third order polynomial equation  $y^3 - y\omega_p^2/(1 + p_{\parallel 0}^2)^{1/2} + i\kappa W = 0$  and the constants  $C_{i,x,y}$  are determined by the initial conditions. It is obvious from the form of the polynomial equation that one of its roots is imaginary and positive, while the other two are complex. The two terms in the solution with complex frequencies describe a damped wave, while the third term corresponds to a spurious, exponentially growing, term that can be excluded by an appropriate choice of the initial conditions.

## VI. CONCLUSION AND DISCUSSIONS

In the present paper we considered the problem of the back reaction of the produced electron-positron pairs on the electromagnetic wave. We showed that there is a loss of wave energy due to the pair production and acceleration of these pairs in the electromagnetic field of the wave.

We studied the propagation of a relativistically strong electromagnetic wave in an underdense electron-positron plasma. As is well known, a plane wave does not produce electron-positron pairs in vacuum. However the situation changes in a plasma. Due to the fact that in plasma the first field invariant  $\mathcal{F}$  does not vanish, a plane wave can produce pairs that back react on the wave. In order to describe the behavior of the electrons and positrons, we adopted a kinetic plasma description and used the relativistic Boltzmann-Vlasov equation. Solving this equation we obtained the distribution functions of electrons and positrons which we used to derive the current density that enters the right-hand side of Maxwell equation for the electric field. We note that all the calculations were carried out in the reference frame moving with the group velocity of the wave. In this frame there is no magnetic field as the wave has only an electric field.

The solutions of the Maxwell equation for the evolution of the electric field in the plasma for different initial amplitudes are shown in Fig. 1. We see that, when the process of electron-positron pair production is taken into account, the evolution of the vector potential in the plasma leads to the damping of the wave, accompanied by a nonlinear shift of its frequency. The damping of the vector potential is due to the pair production which takes away a portion of the wave energy. This process is followed by the increase of plasma density which leads to the up-shifting of the wave frequency.

The damping of the vector-potential amplitude, together with its frequency increase, gives rise to an interesting phenomenon—the decrease of the wave amplitude followed by the increase of its frequency. Pair production decreases the wave amplitude while the growth of plasma density all over the space causes the increase of the wave frequency which is proportional to square root of the density. This effect resembles the case when the electromagnetic wave propagates in an ionizing medium [25].

Since the pair production rate depends on the field amplitude exponentially, an unbalanced damping of the field components occurs and leads to the change of the wave polarization (see Fig. 2). One can clearly see the change of the electric field polarization from circular to elliptic and the decrease of one of the two electric field components.

Finally, we considered two limiting cases in order to identify the properties and the exact causes of this nonlinear behavior of the wave. In the first case the initial amplitude of the electric field is so large that the pair production and the change of the wave properties occur instantaneously. In the second case the pair production rate is small so that the properties of the wave change slowly. In the first case we obtain an electric field oscillating with a new frequency and amplitude. The frequency increase is due to the fact that the density of plasma increases because of the pair production. In the second case we obtain a damped wave with an amplitude that decreases slowly with time. We see that the process of electron-positron pair production leads to the damping of the wave. There is a nonlinear shift of its frequency due to the decrease of its amplitude: as a result the frequency is up-shifted.

## ACKNOWLEDGMENTS

The authors would like to acknowledge fruitful discussions with N. B. Narozhny, V. D. Mur, L. B. Okun, and V. S. Popov. This work was supported in part by the Russian Fund for Fundamental Research under projects 03-02-17348 and 04-02-17157, and the Federal Program of the Russian Ministry of Industry, Science and Technology grant No 40.052.1.1.1112.

## APPENDIX: PROPERTIES OF THE POLARIZATION CURRENT

The form of the polarization current can be derived from the energy-conservation law. In order to calculate the expres-

sion for the polarization current we calculate the second moment of the kinetic equation (12):

$$\begin{aligned} \sum_{\alpha=\pm} \int [(m^2 + p^2)^{1/2} - m] \left\{ \frac{\partial f_\alpha}{\partial t} + e_\alpha \mathbf{E} \cdot \frac{\partial f_\alpha}{\partial \mathbf{p}} \right\} d^3 p \\ = \sum_{\alpha=\pm} \int [(m^2 + p^2)^{1/2} - m] q_\alpha(\mathbf{E}, \mathbf{p}) d^3 p, \end{aligned} \quad (\text{A1})$$

which we write as

$$\frac{\partial K}{\partial t} - \mathbf{j}_{cond} \cdot \mathbf{E} = \Sigma - m \frac{\partial n}{\partial t}, \quad (\text{A2})$$

where

$$K = \sum_{\alpha=\pm} \int [(m^2 + p^2)^{1/2} - m] f_\alpha(\mathbf{p}, t) d^3 p$$

is the kinetic energy of the plasma particles, and

$$\mathbf{j}_{cond} = e \int \frac{\mathbf{p}}{(m^2 + \mathbf{p}^2)^{1/2}} \sum_{\alpha=\pm} e_\alpha f_\alpha(\mathbf{p}, t) d^3 p$$

is the conduction current. The second moment of the right-hand side of the kinetic equation gives two terms. One of them,  $m \partial n / \partial t$ , is related to the rest energy increase due to the pair production, and another is equal to

$$\Sigma = \sum_{\alpha=\pm} \int (m^2 + p^2)^{1/2} q_\alpha(\mathbf{E}, \mathbf{p}) d^3 p.$$

We can thus rewrite Eq. (A2) in the following form

$$\frac{\partial}{\partial t} (K + nm) = \mathbf{j}_{cond} \cdot \mathbf{E} + \Sigma, \quad (\text{A3})$$

where the expression inside the brackets corresponds to the full energy of the plasma particles. The energy balance equation for the electromagnetic field in the moving reference frame where only the electric field is present, can be obtained by multiplying Eq. (19) by the vector  $\mathbf{E}$

$$\frac{\partial}{\partial t} \left( \frac{\mathbf{E}^2}{8\pi} \right) = -\mathbf{j}_{cond} \cdot \mathbf{E} - \mathbf{j}_{pol} \cdot \mathbf{E}. \quad (\text{A4})$$

Here we took into account that no external current is present in our case. Adding Eqs. (A3) and (A4) we obtain

$$\frac{\partial}{\partial t} \left( K + n + \frac{\mathbf{E}^2}{8\pi} \right) = -\mathbf{j}_{pol} \cdot \mathbf{E} + \Sigma. \quad (\text{A5})$$

The expression inside the brackets in Eq. (A5) is the energy of the system of the electromagnetic field and the plasma electrons and positrons. Energy conservation requires the right-hand side of Eq. (A5) to be zero. Thus the polarization current should be of the form

$$\mathbf{j}_{pol} = \frac{\mathbf{E}}{|\mathbf{E}|^2} \sum_{\alpha=\pm} \int (m^2 + \mathbf{p}^2)^{1/2} q_\alpha(\mathbf{p}, t) d^3 p. \quad (\text{A6})$$

- 
- [1] F. Sauter, Z. Phys. **98**, 714 (1931); **98**, 714 (1931).  
[2] W. Heisenberg and H. Z. Euler, Z. Phys. **98**, 714 (1936).  
[3] J. Schwinger, Phys. Rev. **82**, 664 (1951).  
[4] E. Brezin and C. Itzykson, Phys. Rev. D **2**, 1191 (1970).  
[5] V. S. Popov, JETP Lett. **13**, 185 (1971); Sov. Phys. JETP **34**, 709 (1972).  
[6] V. S. Popov, JETP Lett. **18**, 255 (1973); Sov. J. Nucl. Phys. **19**, 584 (1974).  
[7] M. S. Marinov and V. S. Popov, Sov. J. Nucl. Phys. **16**, 449 (1973).  
[8] N. B. Narozhny and A. I. Nikishov, Sov. Phys. JETP **38**, 427 (1974).  
[9] F. V. Bunkin and I. I. Tugov, Dokl. Akad. Nauk SSSR **187**, 541 (1969).  
[10] G. J. Troup and H. S. Perlman, Phys. Rev. D **6**, 2299 (1972).  
[11] T. Tajima and G. Mourou, Phys. Rev. ST Accel. Beams **5**, 031301 (2002).  
[12] D. L. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997).  
[13] S. V. Bulanov, T. Zh. Esirkepov, and T. Tajima, Phys. Rev. Lett. **91**, 085001 (2003).  
[14] S. S. Bulanov, Phys. Rev. E **69**, 036408 (2004).  
[15] S. S. Bulanov, N. B. Narozhny, V. D. Mur, and V. S. Popov, hep-ph/0403163.  
[16] K. Kajantie and T. Matsui, Phys. Lett. **164B**, 373 (1985).  
[17] G. Gatoff, A. K. Kerman, and T. Matsui, Phys. Rev. D **36**, 114 (1987).  
[18] Y. Kluger, J. M. Eisenberg, and B. Svetitsky, Phys. Rev. Lett. **67**, 2427 (1991); Phys. Rev. D **45**, 4659 (1992); Int. J. Mod. Phys. E **2**, 333 (1993); Y. Kluger, E. Mottola, and J. M. Eisenberg, Phys. Rev. D **58**, 125015 (1998).  
[19] S. Schmidt, D. Blanschke, G. Ropke, S. A. Smolyansky, A. V. Prosorkevich, and V. D. Toneev, Int. J. Mod. Phys. E **7**, 709 (1998); Phys. Rev. D **59**, 094005 (1999); J. C. R. Bloch, V. A. Myserny, A. V. Prosorkevich, C. D. Roberts, S. M. Schmidt, S. A. Smolyansky, and V. D. Toneev, *ibid.* **60**, 116011 (1999).  
[20] C. E. Dolby and S. F. Gull, Ann. Phys. (N.Y.) **297**, 315 (2002).  
[21] R. Ruffini, L. Vitagliano, and S. S. Xue, Phys. Lett. B **559**, 12 (2003).  
[22] H. K. Avetissian, A. K. Avetissian, G. F. Mkrtchian, and Kh. V. Sedrakian, Phys. Rev. E **66**, 016502 (2002).  
[23] A. I. Akhiezer and R. V. Polovin, Sov. Phys. JETP **30**, 915 (1956).  
[24] I. V. Smetanin, D. Farina, J. Koga, K. Nakajima, and S. V. Bulanov, Phys. Lett. A **320**, 438 (2004).  
[25] S. C. Wilks, J. M. Dawson, and W. B. Mori, Phys. Rev. Lett. **61**, 337 (1988); A. V. Kim, S. F. Lirin, A. M. Sergeev,

- E. V. Vanin, and L. Stenflo, Phys. Rev. A **42**, 2493 (1990); E. Esarey, G. Joyce, and P. Sprangle, *ibid.* **44**, 3908 (1991); V. B. Gil'denburg, V. I. Pozdnyakova, and I. A. Shereshevskii, Phys. Lett. A **203**, 214 (1995); E. Conejero Jarque, F. Cornolti, and A. Macchi, J. Phys. B **33**, 1 (2000).
- [26] S. P. Kuo, Phys. Rev. Lett. **65**, 1000 (1990); N. Yugami, T. Niiyama, T. Higashiguchi, H. Gao, S. Sasaki, H. Ito, and Y. Nishida, Phys. Rev. E **65**, 036505 (2002).